Buckling of Cold-Formed Thin-Walled Steel Members with Holes

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Abstract: Since the early 1940s, steel structural cold-formed members such as columns, beams and beam-columns with holes had been applied all over the world, since such cold-formed steel members can provide an economical design. In addition, unusual sectional forms can be easily made by the cold forming process. While during the last century, only individual compression or bending members are investigated. The analysis and design of steel cold-formed sections with holes are rather complex especially when the shape of holes and their arrangements are unusual. In this paper, the calculating methods for the flexural-torsional buckling and distortional buckling of cold-formed thin-walled members with and without holes are introduced with a few examples based on theoretical analysis and experimental data.

Keywords: cold-formed thin-walled steel member with holes; flexural-torsional buckling; distortional buckling

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1 Introduction
The cold-formed columns and beams with holes are usually used in some low and multi-story frames. Due to the existence of holes in cold-formed channel columns or beams, the cross-sections of such members are weakened. Therefore, it is necessary to investigate the behavior and failure modes of the cold-formed members with holes.

2 Global Flexural Buckling of Cold-Formed Steel Perforated Columns
Many holes usually are placed in the webs of cold-formed steel framed columns and beams as shown in Fig. 1(1). The cold-formed members without holes may be calculated from the elastic buckling analysis, in which the finite strip method (FSM) are usually used to design according to national codes. However, the discontinuities caused by many holes can not be explicitly modeled with FSM and the formal calculation of the elastic buckling loads requires shell finite element method (SFEM) for these models. From investigation of Moon C D and Schafer B W(2), as shown in Fig. 1. It shows the critical buckling load versus buckling half-wavelength for a cold-formed steel lipped C-section column with FSM analysis and the influence of the slotted holes with SFM eigen-buckling analysis.
Using the column coordinate system as shown in Fig. 2, the approximate equation of the elastic flexural buckling load $P_{cr}$ may be obtained from Rayleigh-Ritz energy solution. For a simply supported perforated column, its elastic flexural buckling deflection may be assumed as follows:

$$y = A \sin \pi x / l$$  \hspace{1cm} (1)

Let $\delta(U + V)$ be the summation of the variation in the total potential energy-in which $U$ is the strain energy in the column due to bending and $V$ is the potential energy of the external load, which is equal to the external work in magnitude but opposite in sign by the axial load $P$ on the column, as it shortens axially due to bending. The equilibrium condition in the column bent state is as follows:

$$\delta(U + V) = 0$$  \hspace{1cm} (2)

The elastic strain energy of the perforated column is:

$$U = \frac{1}{2} \int \int E I(\pi x / l)^2 dx$$  \hspace{1cm} (3)

If the moment of inertia $I(x)$ of the perforated column varies discretely as shown in Fig. 2, between the gross section $I_{g}$ and the net cross-section $I_{n}$, the bending strain energy of the perforated column is:

$$U = \frac{PA^2 \pi^2}{2l} \left[ \sum_{i=1}^{n} \int_{x_i}^{x_{i+1}} \sin \left( \frac{\pi x}{l} \right) dx \right] +$$

$$\sum_{i=1}^{n} \int_{x_i}^{x_{i+1}} \cos \left( \frac{\pi x}{l} \right) dx$$  \hspace{1cm} (4)

where $x_0 = 0$, $x_1 = x_1 - 0.5l_{hole}$, $x_{i} = x_{i-1} + 0.5l_{hole}$, $x_{i+1} = x_{i-1} + 0.5l_{hole}$, and $x_{i+1} = l$.

The distance from the end of the column to the centerline of the hole $j$ is $x_{j}$ and $l_{hole}$ is the length of hole $j$ for $j = 1, 2, \ldots, n$ holes.

The potential energy of external load $P$ is approximated as:

$$V = - \frac{P}{2} \int \left( \frac{dy}{dx} \right)^2 dx = - \frac{PA^2 \pi^2}{2l} \int \left( \cos \left( \frac{\pi x}{l} \right) dx \right) =$$

$$- \frac{PA^2 \pi^2}{4l}$$  \hspace{1cm} (5)

For the single deformed column only in one-half-sin-wave, Eq. (2) may be rewritten as:

$$\frac{dU}{dA} + \frac{dV}{dA} = 0$$  \hspace{1cm} (6)

The elastic buckling load for an arbitrarily spaced net-section region is:

$$P_{cr} = \frac{\pi^2 E}{l^2} \left[ \frac{I_{g}}{l} + l_{n} \right]$$  \hspace{1cm} (7)

in which the total length of column with holes is:

$$l_{n} = \sum_{i=1}^{n} l_{hole,i}$$  \hspace{1cm} (8)

$$T = \frac{l}{2 \pi} \sum_{i=1}^{n} \cos \left( \frac{2\pi l_{hole,i}}{l} \right) \sin \left( \frac{\pi l_{hole,i}}{l} \right)$$  \hspace{1cm} (9)

If the holes are spaced symmetrically about the longitudinal midline of the column, that is $x_{-1} = l/2$, then $T = 0$ and Eq. (7) becomes:
3. Flexural-Torsional Buckling of Cold-Formed Steel Perforated Beam

For the simply supported beam without holes under uniform bending moment $M_0$, its equilibrium differential equations are:

$$EI_{e} \frac{d^4 u}{dx^4} + M_e \varphi = 0$$  \hspace{1cm} (11)

$$EI_{e} \frac{d^4 \varphi}{dx^4} - (2\beta) \frac{M_0}{l_0} \frac{d^2 \varphi}{dx^2} + M_0 \frac{d^2 \varphi}{dx^2} = 0$$  \hspace{1cm} (12)

in which $\beta$ is the unsymmetrical section flexural constant.

For the beam with holes, in Eq. (11) and (12), using weighted average approach, these calculations of warping moment of inertia $I_{e,\varphi}$, unsymmetrical section constant $\beta_{e,\varphi}$ and torsional moment of inertia $I_{e,\varphi}$ all may be transformed into weighted average values.

The symmetrical t-section with sectional height $h_s$, the value of $\beta_{e,\varphi}$ is equal to zero,

$$I_{e,\varphi} = I_{e,\varphi} h_s / 4$$  \hspace{1cm} (13)

$$I_{e,\varphi} = \frac{I_1 + I_2}{l}$$  \hspace{1cm} (14)

Referring to the section as shown in Fig. 1, the torsional moment of inertia $I_{e,\varphi}$ of the beam with holes is:

$$I_{e,\varphi} = \frac{2}{3} \left[ \frac{b}{t} \left( \frac{h_s - h_b}{2} \right) t_s \right]$$  \hspace{1cm} (15)

$$I_{e,\varphi} = \frac{I_1 + I_2 + I_{e,\varphi}}{l}$$  \hspace{1cm} (16)

The elastic flexural-torsional buckling moment of the cold-formed steel perforated beam is:

$$M_{e,\varphi} = \frac{\pi}{T} \sqrt{EI_{e,\varphi} \left( GL_{e,\varphi} + EI_{e,\varphi} \varphi \right)}$$  \hspace{1cm} (17)

4. Distortional Buckling Load of Cold-Formed Steel Perforated Columns and Beams

A shell finite element analysis method may be used to study the influence of web holes on the web stiffness of lipped C-section cold-formed steel columns and beams. A simplified method is to reduce the transverse bending stiffness from the web with holes. The approximate thickness $t_{web,hole}$ in web hole is:

$$t_{web,hole} = (1 - t_{col}/l_{col})^{1/3} t_{col}$$  \hspace{1cm} (18)

in which $t_{col}$ is the distortional buckling half-wavelength taken from Schafer S W and Pekoz T handle method. To calculate $P_{col}$ including the influence of holes, first a finite strip method (FSM) is made with the gross cross-section to identify the distortional half-wavelength $l_{col}$. Then, the web thickness will be modified from Eq. (18). Including the influence of holes, the distortional buckling load $P_{col}$ of cold-formed may be obtained with another FSM or with Schafer S W and Pekoz T handle method.

5. Local Buckling Load of Cold-Formed Steel Perforated Columns and Beams

According to the study of Maiorana E, Pellegrino C and Modena C(2) with FSM, a modified local buckling coefficient $k$ may be used to calculate the local buckling load of the perforated plates under axial compression and bending moment. Some practical design formulas for estimation of the modified local buckling coefficients, depending on the stress ratio $\psi$, various plate aspect ratios $a/b$, hole dimension ratios $d/b$ and hole location distances $x$ are proposed. These practical design formulas are developed both for circular and rectangular perforated plates with RS holes (with the major dimension $l_{hole}$ parallel to the vertical plate z-axis) and with RL holes (with the major dimension $l_{hole}$ parallel to the horizontal plate x-axis) as shown in Fig. 3 (a) and (b).

![Fig. 3 Static scheme of perforated plates under axial compession and bending moment](image)

(a) Perforated with circular hole, (b) Perforated with rectangular hole
Fig. 4  Modified buckling coefficient of plate $k$ vs. stress ratios $\psi$ for circular holes having center nodal point

(a) $a/b=2$,  (b) $a/b=3$

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$a/b$</th>
<th>$a/b=2$</th>
<th>$a/b=3$</th>
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<td>$k=10.2\phi + 8.0$</td>
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<td>$k=3.4\phi + 8.0$</td>
<td>$k=3.4\phi + 7.7$</td>
</tr>
</tbody>
</table>

Fig. 5  Modified buckling coefficient of plate $k$ vs. stress ratios $\psi$ for circular holes having center in maximum point

(a) $a/b=1$,  (b) $a/b=2$,  (c) $a/b=3$

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$a/b$</th>
<th>$a/b=1$</th>
<th>$a/b=2$</th>
<th>$a/b=3$</th>
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<td>0.1</td>
<td>$k=16.4\phi + 7.3$</td>
<td>$k=159.0\phi + 6.8$</td>
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</tr>
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<td>$&lt; 0$</td>
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<td>$k=12.7\phi + 6.1$</td>
<td>$k=135.0\phi + 6.8$</td>
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<td></td>
<td>0.5</td>
<td>$k=7.0\phi + 5.4$</td>
<td>$k=10.6\phi + 6.8$</td>
<td>$k=10.7\phi + 6.9$</td>
</tr>
</tbody>
</table>

These modified buckling coefficients of perforated plates are shown in Figs. 4 - 9. It is noticed that for the stress ratios $\psi \geq 0$, these curves for each hole diameter ($d/b = 0, 1, 0, 3$ and 0, 5) become coincident. Since the modified local buckling coefficients of perforated plates under axial compression and bending moment may be greater or less than the local buckling coefficients of plates without holes. The local buckling load of cold-formed steel columns and beams is as follows:

$$P_{lt} = \min (P_{lt,\text{buck}}, P_{lt,\text{colb}})$$

in which, $P_{lt,\text{buck}}$ is the local buckling load of the gross section and $P_{lt,\text{colb}}$ is the local buckling load of the cross-section with holes.
Fig. 6  Modified buckling coefficient of plate $k$ vs. stress ratios $\psi$ for RS holes having center in nodal point
(a) $a/b=2$, (b) $a/b=3$

<table>
<thead>
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</tr>
<tr>
<td>0.5</td>
<td>$k=13.6\psi+10.1$</td>
<td>$k=15.1\psi+7.9$</td>
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</tr>
<tr>
<td>$\phi\geq0$</td>
<td>-</td>
<td>$k=-4.2\psi+8.8$</td>
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</tbody>
</table>

Fig. 7  Modified buckling coefficient of plate $k$ vs. stress ratios $\psi$ for RS holes having center in maximum point
(a) $a/b=1$, (b) $a/b=2$, (c) $a/b=3$

<table>
<thead>
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<th>$\phi$</th>
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<tr>
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<td>$k=-8.4\psi+9.6$</td>
<td>$k=-15.7\psi+7.5$</td>
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<td></td>
</tr>
<tr>
<td>$\phi\geq0$</td>
<td>-</td>
<td>$k=-3.5\psi+7.8$</td>
<td>$k=-3.7\psi+7.5$</td>
<td>$k=-3.7\psi+7.4$</td>
</tr>
</tbody>
</table>
Fig. 8  Modified buckling coefficient of plate $k$ vs. stress ratios $\psi$ for RL holes having center in nodal point
(a) $a/b=2$, (b) $a/b=3$

![Diagram showing modified buckling coefficient for different stress ratios and plate configurations.]

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$\phi$</th>
<th>$a/b$ = 2</th>
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</thead>
<tbody>
<tr>
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<td>$k = -15.2 \phi + 7.5$</td>
<td>$k = -15.4 \phi + 7.4$</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>$k = -10.2 \phi + 7.5$</td>
<td>$k = -10.4 \phi + 7.4$</td>
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<tr>
<td></td>
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<td>$k = -6.5 \phi + 7.5$</td>
<td>$k = -6.7 \phi + 7.4$</td>
</tr>
<tr>
<td>$\phi \geq 0$</td>
<td>-</td>
<td>$k = -3.4 \phi + 7.5$</td>
<td>$k = -3.4 \phi + 7.4$</td>
</tr>
</tbody>
</table>

Fig. 9  Modified buckling coefficient of plate $k$ vs. stress ratios $\psi$ for RL holes having center in maximum point
(a) $a/b=1$, (b) $a/b=2$, (c) $a/b=3$

![Diagram showing modified buckling coefficient for different stress ratios and plate configurations.]

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$\phi$</th>
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<td>$k = -15.1 \phi + 7.3$</td>
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<tr>
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<td>$k = -8.3 \phi + 5.2$</td>
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<td>$k = -11.6 \phi + 6.0$</td>
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<tr>
<td>$\phi \geq 0$</td>
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<td>$k = -2.6 \phi + 5.2$</td>
<td>$k = -2.9 \phi + 5.9$</td>
<td>$k = -2.9 \phi + 5.9$</td>
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For the research to develop a direct experiment to verify the strength reduction caused by the web holes, Lue D M, Chung P T, Liu J L, et al. \(^{[1]}\) tested a total of 21 channel specimens including cold-formed steel columns with and without holes as shown in Fig. 10 and Fig. 11. The failure mode of all these specimens is observed and identified as flexural-torsional buckling type. The experimental results indicate that the average strength is slightly reduced by the web holes and show the finite element method (FEM) is an efficient tool to calculate the strength of qualified specimens.
For the sake of observing and quantifying the relationship between the elastic buckling behavior and the tested response of the perforated cold-formed steel columns, a testing research program had been conducted by Moen C D and Schafer B W. Compression tests were conducted on 24 short and intermediate length cold-formed steel columns with and without slotted holes. For each specimen, a shell finite element eigen buckling analysis was also conducted, such that the influences of the boundary conditions, the hole locations and the plate with hole on elastic local distortional and global flexural buckling responses could be captured. Experiments on perforated cold-formed steel columns show that, the slotted web holes are shown to have a minimal influence on the tested ultimate strength in the specimens considered and only the post-peak ductility is decreased in some cases. Figs. 12 and 13 show the comparisons of the load-displacement response for the short and intermediate length column specimens with and without holes respectively.

Example 1 Using Schafer B W and Pekoz T handle method, determine the elastic distortional buckling compression load of the perforated cold-formed steel column simply supported with a lateral support at its mid-span as shown in Fig. 14. The properties of steel are, \( f_y = 31 \text{ kN} \cdot \text{cm}^{-2} \), \( E = 20340 \text{ kN} \cdot \text{cm}^{-2} \) and \( G = 900 \text{ kN} \cdot \text{cm}^{-2} \). The section area shown in Fig. 14 (a) is \( A = 4.2 \text{ cm}^2 \).
section is equal to \( t_{ml} = 56.93 \text{ cm} \). Its elastic distortional ultimate load of column without holes is \( P_{pl,n,h} = 103.11 \text{ kN} \cdot \text{cm}^2 \).

**Solution:**

1) Determine the modified web thickness \( t_{ml,n,h} \) of modified section,

\[
A_v = \pi (b + d) = 0.15 \times (7.45 + 1.525) = 1.346 \text{ cm}^2
\]

\[
x_{cf} = \frac{b}{2(b + d)} = \frac{7.45}{2 \times (7.45 + 1.525)} = 0.092 \text{ cm}
\]

\[
x_{cf} = \frac{d^2}{2(b + d)} = \frac{1.525^2}{2 \times (7.45 + 1.525)} = 0.129 \text{ cm}
\]

\[
h_{cf} = (b - x_{cf}) = (7.45 - 0.092) = 7.358 \text{ cm}
\]

\[
x_{cf} - h_{cf} = b = 7.45 \text{ cm}, \quad h = 10.05 \text{ cm}
\]

\[
I_v = \frac{t (b^3 + 4bd^3 + t^2 bd + d^4)}{12(b + d)} = 0.15 \times (0.15^3 + 7.45^3 + 4 \times 1.525 \times 7.45^3 + 0.15^2 \times 7.45 \times 1.525 + 1.525^3) = 0.1568 \text{ cm}^4
\]

\[
I_{cf} = \frac{\pi d^4}{4(b + d)} = 1.525 \times 7.45^2 \times 1.525^2 \times 7.8034 \text{ cm}^4
\]

\[
I_v = \frac{b^4 d}{4(b + d)} = \frac{7.45^4 \times 1.525^2}{4 \times (7.45 + 1.525)} = 0.5393 \text{ cm}^4
\]

\[
I_{cf} = \frac{\pi d^4}{4(b + d)} = \frac{7.45^4 \times 1.525^2}{4 \times (7.45 + 1.525)} = 0.5393 \text{ cm}^4
\]

\[
L_v = \frac{t (b + d)}{3} = 0.15 \times (7.45 + 1.525)/3 = 0.0101 \text{ cm}, \quad L_{cf} = 0
\]

(2) Determine the distortional buckling load.

1) The critical half-wavelength \( \lambda_c \) of the compression member with web holes, elastic and geometrical rotational stiffness of flange \( k_{s,fl} \), \( k_{s,fl} \), and web \( k_{s,we} \), \( k_{s,we} \).

\[
\lambda_c = \left[ \frac{6 \pi h (1 - \nu^2)}{P_{pl,h}} \right] \left[ I_v (x_{cf} - h_{cf})^3 + I_{cf} - \left( \frac{1 - t_{ml,n,h}}{l_{ml,n,h}} \right)^{1/3} l_{ml,n,h} \right]
\]

\[
l_{ml,n,h} = (1 - \frac{t_{ml,n,h}}{l_{ml,n,h}})^{1/3} l_{ml,n,h} = (1 - \frac{6}{56.93})^{1/3} \times 0.15 = 0.1445 \text{ cm}
\]

2) Determine the elastic distortional buckling load of perforated column.

(1) According to Fig. 14 (a), determine the geometrical properties of compression flange.

2. The compressive strain at the elastic buckling limit of the column is

\[
\sigma_{pl} = \frac{6 \pi h (1 - \nu^2)}{P_{pl,h}} \left( \frac{1}{l_{ml,n,h}} \right) \left( \frac{1}{l_{ml,n,h}} \right)^{1/3} l_{ml,n,h}
\]

\[
\sigma_{pl} = 4.2 \times 23.394 = 98.26 \text{ kN}
\]

\[
P_{pl,h} = \frac{A \sigma_{pl}}{P_{pl,h}} = 98.26 / 103.11 = 0.9529 \text{ kN}
\]
The reduction factor is 0.9529 or drops 4.71%.

**Example 2** Using Schafer B W and Peköz T handle method, determine the elastic distortional buckling bending moment of the perforated cold-formed steel beam simply supported with a lateral support at its mid-span as shown in Fig. 15. The properties of steel are: \( f_r = 31 \text{ kN} \cdot \text{cm}^{-1} \cdot \text{cm}^{-2}, E = 20340 \text{ kN} \cdot \text{cm}^{-1} \text{ and } G = 7900 \text{ kN} \cdot \text{cm}^{-2} \). The section dimensions area shown in Fig. 15 (a), \( A = 5.49 \text{ cm}^2 \), \( J = 338.71 \text{ cm}^4 \), \( W = 34.13 \text{ cm} \cdot I = 34.76 \text{ cm}^4 \), \( l_0 = 0.441 \text{ cm} \cdot l = 2.67281 \text{ cm}^3 \). The distortional half-wavelength \( l_{cd} \) with gross-section is equal to \( l_{cd} = 58.14 \text{ cm} \). The elastic distortional buckling bending moment of beam without holes is \( M_{cr, hole} = 84.93 \text{ kN} \cdot \text{cm} \) by Schafer B W and Peköz T method(1).

**Solution**

1) Determine the modified web thickness \( t_{w,hole} \) of modified section.

\[
R_{w,hole} = \left(1 - \frac{t_{hole}}{t_{w}}\right)^{1/3} \quad t_{w,hole} = \left(1 - \frac{6}{58.14}\right)^{1/3} \times 0.15 = 0.144 \text{ cm}
\]

2) Determine the elastic distortional buckling bending moment of the perforated beam.

(1) According to Fig. 15 (d), determine the geometrical properties of compression flange.

\[
A_f = (6.85 + 1.25) \times 0.15 = 1.256 \text{ cm}^2
\]

\[
x_f = \frac{2b^3}{2(b + d)} = \frac{2 \times (6.85 + 1.25)}{2 \times (6.85 + 1.525)} = 2.801 \text{ cm}
\]

\[
y_f = -\frac{d}{2(b + d)} = -\frac{1.525}{2 \times (6.85 + 1.525)} = -0.138 \text{ cm}
\]

\[
h_f = -(b - x_f) = -(6.85 - 2.801) = -4.048 \text{ cm}
\]

\[
I_{f} = \frac{(b^3)(b + 4d)}{12(b + d)} = 0.15 \times \left(6.85 + 0.15 \times \frac{(b + 1.525) + (4 \times 6.84 + 1.525 \times 1.525)}{12 \times (6.85 + 1.525)}\right) = 0.155 \text{ cm}^4
\]

\[
I_{j} = \frac{8b^4(6.85 + 1.525)}{12(6 + d)} = 0.15 \times 6.85 \times (6.85 + 1.525) = 6.2125 \text{ cm}^4
\]

\[
I_{cs} = \frac{8b^4d}{4(6 + d)} = 0.15 \times 6.85 \times 1.525 \times 6.85 + 1.525 \times 1.525) = 0.448 \text{ cm}^4
\]

\[
I_{cr} \approx 0
\]

\[
I_{cr} = (b^3(x_f - h_f))^3 = 0.15 \times (6.85 + 1.525) = 0.09 \text{ cm}^4
\]

2) Determine the elastic distortional buckling bending moment of the perforated beam.

(1) The elastic half-wavelength \( l_0 \) of the beam with web hole, elastic and geometrical rotational stiffness are \( k_{y,cr} \), \( k_{x,cr} \), \( k_{y,x} \), \( k_{x,x} \).

\[
l_0 = \frac{4 \pi^2 h (1 - \nu^2) [I_{cr} (x_f - h_f)^2 + I_{cr} - I_{cr} (x_f - h_f)^2 / I_{cr}] + 4 \pi^2 h (1 - \nu^2)}{0.144 \times 6.85 + 1.525} = \frac{4 \pi^2 h (1 - \nu^2)}{0.144 \times 6.85 + 1.525}
\]
6 Conclusions

Two design formulas to predict the ultimate strength capacities of steel cold-formed columns and beams containing multiple openings have been given in which the cold-formed channel sections with different web plate slenderness ratio, opening types, opening sizes are considered. The theoretical analysis and experimental investigation are presented. Two examples are used to compare the calculating results for the cold-formed channel sections with and without holes.

参考文献:
[3] Schafer B W and Pekoz T, Direct strength prediction of cold-formed steel members using numerical elastic buckling solutions[C]/14th International Speciality Conference on Cold-Formed Steel Structures, St. Louis Missouri, 1998: 69-76

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